LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

SECOND SEMESTER – APRIL 2012

# MT 2811 - MEASURE THEORY AND INTEGRATION

Date : 19-04-2012 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

**ANSWER ALL QUESTIONS:-**

1. (a) State and prove countable sub additive theorem for outer measures. **(5)**

(OR)

(b) Prove that every interval is measurable. **(5)**

(c) Prove that there exists a non measurable set. **(15)**

(OR)

(d) Show that Lebesgue measure is regular. **(15)**

1. (a) Let f and g be non negative measurable functions. Then prove ∫ f dx + ∫ g dx = ∫ (f + g) dx . **(5)**

(OR)

(b) Prove that if the sequence  is a sequence of non-negative measurable function

then . **(5)**

(c) State and prove Lebesgue Dominated Convergence theorem. **(15)**

(OR)

(d) If f is Riemann integrable and bounded over the finite interval [a,b] then prove that f

is integrable and . **(15)**

1. (a) Show that with a usual notations the outer measure μ\* on H(ℜ),and the **(5)**

outer measure defined by  on S( ℜ) and on contains are the same.

(OR)

(b) Prove that if μ\* is an outer measure on H(ℜ), defined by μ on H(ℜ) then contains

, the  -ring generated by ℜ. **(5)**

(c) Show that if  is a measure on a -ring  then the class of sets of the form

for any sets E,N such that While N is contained in some set in of zero

measure is a -ring and the set function defined by is a

complete measure on . **(15)**

(OR)

(d) Prove that if is an outer measure on H(ℜ),. Let  denote the class of 

Measurable sets then Prove that is a - ring and restricted to is a complete

measure. **(15)**

1. (a) State and prove Holder’s inequality. **(5)**

(OR)

(b) Define the following terms: convergence in measure, almost uniform convergence and uniform convergence almost everywhere. **(5)**

(c) Let [X, S, ] be a measure space with . If  is convex on (*a*, *b*) where  and *f* is a measurable function such that , for all *x*, prove that . When does equality occur? **(15)**



(OR)

(d) State and prove completeness theorem for convergence in measure. Show that if almost uniform then in measure and almost everywhere. **(15)**



1. (a) Define a positive set and show that a countable union of positive sets with respect to a

signed measure *v* is a positive set. **(5)**

(OR)

(b) Let *v* be a signed measure and let  be measure on [X, S] such that  are - finite, «, « then prove that . **(5)**

(c) Let *v* be a signed measure on [X, S]. (i) Let S and . Can you construct a positive set *A* with respect to *v*, such that and ? Justify your answer. (ii) Construct a positive set *A* and a negative set *B* such that . **(15)**



(OR)

(d) State and prove Lebesgue decomposition theorem. **(15)**

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